YOUR NAME: ______________________ ID:_s___________ DOB: ____/____/19__

Department/Program: TN / TBK / BMT / …

Final Exam
Principles of Measurement Systems
(PMS105E.2005-2006.2)
Tuesday, February 7, 2006 (9:00-12:00)

Please write your name, student ID number and date of birth on this page, only your name on all subsequent pages, and number the pages. Hand in all paperwork, including these pages and any draft/scratch pages.
This is not an open-book exam, so please remove all other documents.

Read carefully. Pay attention to units. A numerical result without, or with wrong units, will be considered incorrect. You may assume that I know the answer to the questions posed; Therefore, give derivations and/or motivate your answers as appropriate! If you cannot answer the first part of a question, make a (educated) guess, and continue with the rest… Success!

Question 1

A thermocouple giving a DC output voltage E_{th}=6 mV is connected to a digital voltmeter with input impedance R_L=10 MOhm through a cable with a finite resistance. Both the thermocouple and the digital voltmeter are capacitively coupled to different ground potentials. The equivalent circuit is given in Figure 2.1.

a) Calculate the RMS values of series mode and common mode interference voltages at the voltmeter input.

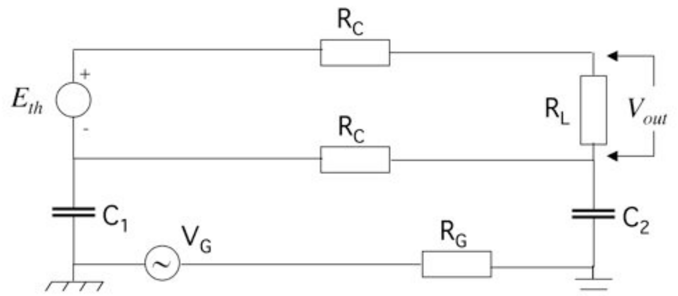

Figure 2.1. Equivalent circuit of the thermocouple – digital voltmeter arrangement. E_{th}=6 mVdc, R_C=100 Ohm, R_L=10 MOhm, R_G=10 Ohm, C_1=100 pF, C_2=1000 pF, V_G=230 Vac (rms).

Question 2

A researcher needs to detect a 3 kHz sinusoidal signal wave $m(t)=m_0+m_1\sin(2\pi f_m t)$ from an optical experiment in a noisy environment. In order to shift the detection bandwith to a higher frequency he considers the use of an amplitude modulation (AM) detection technique. He is able to modulate his experiment at a frequency of 512 kHz.

a) Make a qualitatively correct graph of (the interesting part of) the frequency spectrum.

b) Derive an equation for the ratio of the power in one of the sidebands to the power in the carrier.

It turns out that *1/f* noise is still limiting the sensitivity of the experiment. Therefore, a colleague of the researcher decides to use frequency modulation (FM) instead. Using an Electro-Optical Modulator (EOM), she is able to modulate the experiment at a frequency of 1 GHz, truly beyond the highest frequencies in the noise spectrum.

c) Assuming a low modulation index (defined as the ratio of the maximum frequency deviation of the carrier to the modulation frequency) k=1, make a qualitatively correct graph of (the interesting part of) the frequency spectrum.
d) Explain how the graph of c) changes when the modulation index k is increased from 1 to 10.

Question 3

A force measurement system consists of a piezoelectric crystal, charge amplifier, and recorder with an over-all transfer function given by:

$$G(s) = \frac{\tau s}{1+\tau s} \cdot \frac{1}{\left(\frac{s}{\omega}\right)^2 + \frac{2\xi}{\omega} s + 1}$$

where the charge amplifier time constant equals 0.2 sec, and the natural frequency of the recorder is 60 rad/sec, while its damping ratio is 0.1.

a) Calculate the system dynamic error corresponding to the force input signal given by:
$$F(t) = 25\left\{\sin(20t) + \tfrac{1}{3}\sin(60t)\right\}$$
b) Explain which system modifications are necessary to reduce the system dynamic error.

Question 4

A pressure transducer (manufactured by Baratron) consist of a 0 to 1 bar pressure sensor with integrated current transmitter. Its output is interpreted by a microprocessor system with integrated readout unit. A schematic representation is given in Figure 4.1

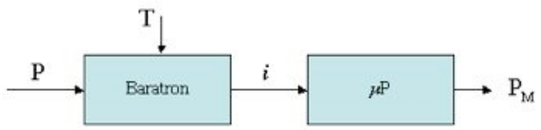

Figure 4.1

The two blocks are described by the following equations:

$$i = a + b \cdot P + c \cdot P^2 + d \cdot (T - T_0) \cdot P$$

$$P_M = \alpha + \beta\sqrt{1 + \gamma \cdot i}$$

The mean values of the parameters and their standard deviations are given in Table IV.

Table IV. System parameter mean values and standard deviations

a = 4 mA	σ_a = 3 10^{-3} mA
b = 11 mA/bar	σ_b = 5 10^{-3} mA/bar

$c = 5$ mA/bar^2	$\sigma_c = 10^{-4}$ mA/bar^2
$d = 0.5$ mA/(ºC bar)	$\sigma_d = 10^{-3}$ mA/(ºC bar)
$T_0 = 21$ ºC	$\sigma_T = 0$ ºC
$\alpha = -1.1$ bar	$\sigma_\alpha = 10^{-4}$ bar
$\beta = 0.64$ bar	$\sigma_\beta = 5\ 10^{-4}$ bar
$\gamma = 0.5$ mA^{-1}	$\sigma_\gamma = 10^{-3}$ mA^{-1}

a) Determine the accuracy and the precision of the pressure reading if the input pressure equals 0.75 bar. The environmental temperature is has an average value of $\langle T \rangle = 21$ ºC, but shows fluctuations characterized by a normal distribution with σ equal to 0.1 ºC.
b) Is the temperature a modifying or interfering input? Explain your answer!
c) Discuss strategies to improve the accuracy and precision of this device.
d) Determine values for the parameters α, β, and γ, that optimize the accuracy of the pressure sensor (i.e., the response of the overall sensor should approach the ideal straight line $P_M = P$ as close as possible; the temperature is 21 ºC).

Bonus Question 5 (free after an example from Dr. Hasper's legacy)
A student has built a sensor to collect 63.7 MHz NMRI signals inside the human body. The signals are brought outside the body to a signal-conditioning unit by an optical link. The signal-conditioning unit consists of three elements, schematically represented in Figure 5.1: *i)* a photodiode which generates a current at the NMR frequency, and its associated shunt resistance (150 Ohm) and capacitance (3 pF), *ii)* a cable 50 cm long representing a capacitance of 100 pF, and *iii)* a current-to-voltage conversion unit consisting of a 10 kOhm resistor.

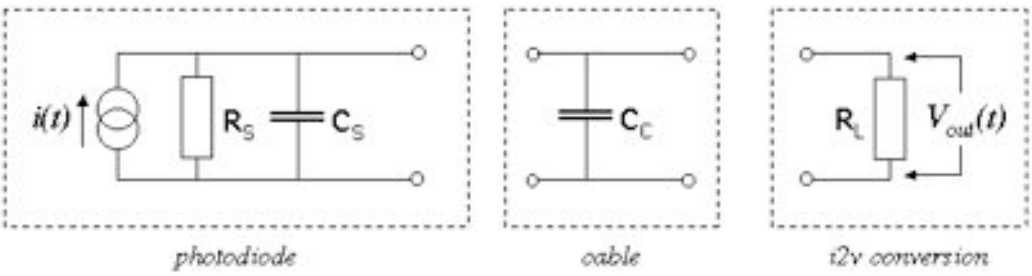

Figure 5.1

a) Determine the transfer function $G(s)=\Delta V_{out}(s)/_D i(s)$ of the combined system *(i to iii)*, and evaluate it numerically at the NMR frequency.
b) Qualitatively graph the amplitude ratio in the range $\log(\omega) = -3$ to $+3$. Identify the important switch-over frequency(-ies) (Note: the logarithmic frequency axis is thus necessarily quantitative, but the units on the vertical axis may be arbitrary).
c) By how many dBs has the gain decreased (with respect to DC) at the NMR frequency?

The student in question decides to replace the current-to-voltage converter of Figure 5.1 by the op-amp circuit of Figure 5.2. $C_A = 10$ nF, $C_F = 100$ pF, $R_F = 100$ kOhm.

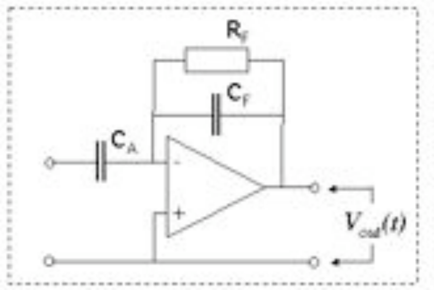

Figure 5.2

d) Determine the transfer function $G(s)=\Delta V_{out}(s)/_{D}i(s)$ of the new combined system and evaluate it numerically at the NMR frequency.
e) Sketch the amplitude ratio in the range log(ω) = -3 to +3.
f) What may be the advantage of using this system over that of Figure 5.1?

… End of the Exam. Please take some time to fill out the questionnaire corresponding to your department or program of study (T)N or TBK.

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Q1

$$V_{CM} = \frac{Z_{C_2}}{Z_{C_2} + R_G + Z_{C_1} + R_C} \cdot V_G$$

$$Z_{C_1} = \frac{1}{j\omega C_1} = \frac{1}{j2\pi f C_1} = \frac{-j}{2\pi\, 50 . 100\, 10^{-12}} = -\frac{j}{\pi} \cdot 10^8$$

$$= -j \cdot 3.18 \cdot 10^7$$

$$Z_{C_2} = -j \cdot 3.18\ 10^6$$

5 $$V_{CM} = \frac{-j\ 3.18\ 10^6 \cdot V_G}{110 - j \cdot 3.18\ 10^6 - j\ 3.18\ 10^7} \simeq \frac{V_G}{11} = 0.091\ V_G$$

$$= 20.9\ V_{RMS}$$

$$V_{SM} = \frac{R_C}{R_C + Z_{C_2} + R_G + Z_{C_1}} \cdot V_G$$

$$= \frac{100}{110 - j \cdot 3.18\ (1.1 \cdot 10^7)} V_G \approx j \cdot \frac{10^2\ V_G}{3.50\ 10^7} = j \cdot 2.86 \cdot 10^{-6} \cdot V_G$$

5 $\rightarrow |V_{SM}| = 2.86\ 10^{-6} \cdot 230\ V_{RMS} = 6.58\ 10^{-4}\ V_{RMS}$

Q2 $m(t) = m_0 + m_1 \sin(\omega_m t)$

$\omega_m = 2\pi f_m$
$\omega_m = 18.8$ ~~krad~~/sec
$= 18800 \ \frac{rad}{sec}$

carrier:

$$C(t) = C_1 \cdot \sin(\omega_c \cdot t)$$

b) AM modulated wave:

$$M(t) = C_1 \cdot \sin(\omega_c t) \{ m_0 + m_1 \sin(\omega_m t) \}$$

$$= m_0 C_1 \cdot \sin \omega_c t + m_1 C_1 \sin(\omega_c t) \cdot \sin(\omega_m t)$$

$$= C_1 \left\{ m_0 \sin \omega_c t + \frac{m_1}{2} \cdot \cos(\omega_c - \omega_m)t - \frac{m_1}{2} \cos(\omega_c + \omega_m)t \right\}$$

→ amplitude of one sideband : $\frac{C_1 m_1}{2}$
" " carrier : $C_1 m_0$

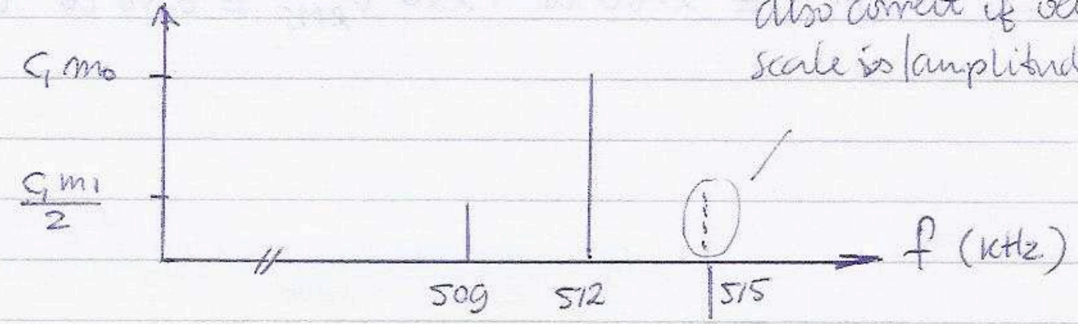
⇒ power ~ amplitude squared

2 → $\frac{P_{1S}}{P_C} = \frac{m_1^2}{4 m_0^2}$

a)

3

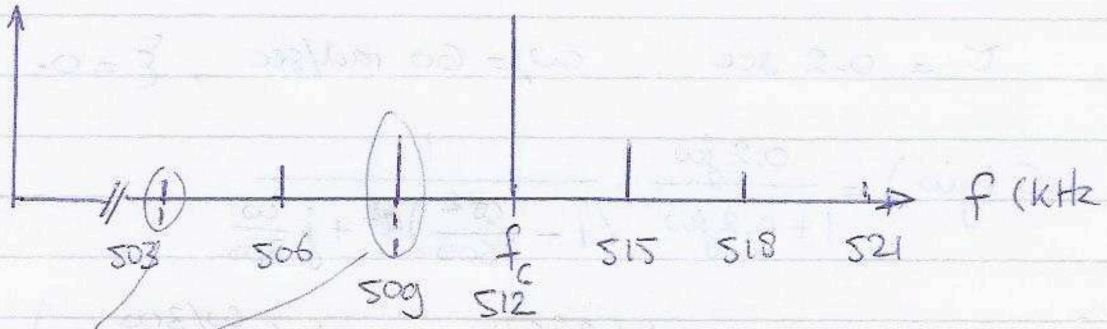
also correct if vertical scale is |amplitude|.



Q2.1

c)

3



positive or negative are "Ok", depending on definition of y-axis.

d) The intensities of the sidebands will change, as well as that of the carrier. Most likely the intensity in the sidebands will show a maximum for $n > 1$ and ~~some~~ sidebands may be stronger than the carrier.

2

Q3

$$G(s) = \frac{\tau s}{1+\tau s} \frac{1}{\left(\frac{s}{\omega_n}\right)^2 + \frac{2\xi s}{\omega_n} + 1}$$

$\tau = 0.2$ sec , $\omega_n = 60$ rad/sec , $\xi = 0.1$

$$G(j\omega) = \frac{0.2j\omega}{1+0.2j\omega} \cdot \frac{1}{\left(1 - \frac{\omega^2}{3600}\right) + j\frac{\omega}{300}}$$

$$\arg(\omega) = \frac{\pi}{2} - \tan^{-1}\left(\frac{0.2\omega}{1}\right) - \tan^{-1}\left(\frac{\omega/300}{1-\omega^2/3600}\right)$$

$$F(t) = 25\left\{\sin(20t) + \frac{1}{3}\sin(60t)\right\}$$

$\underline{\omega = 20}$

$$G(j\cdot 20) = \frac{4j}{1+4\cdot j} \frac{1}{\left(1 - \frac{400}{3600}\right) + \frac{j}{15}}$$

(1) $$|G(j\cdot 20)| = \frac{4}{\sqrt{17}} \cdot \frac{1}{\sqrt{\left(\frac{8}{9}\right)^2 + \left(\frac{1}{15}\right)^2}} = 1.0884$$

(1) $$\arg(20) = \frac{\pi}{2} - \tan^{-1}(4) - \tan^{-1}\left(\frac{\cancel{0.1333}\ 0.06667}{0.8889}\right)$$
$$= 1.571 - 1.326 - 0.07489$$
$$= 0.170 \qquad (= 9.753^\circ)$$

$\underline{\omega = 60}$

$$G(j\cdot 60) = \frac{12j}{1+12j} \cdot \frac{1}{(1-1) + j\cdot\frac{1}{5}}$$

(1) $$|G(j\cdot 60)| = \frac{12}{\sqrt{145}} \cdot \frac{1}{(1/5)} = 0.9965\cdot 5 = 4.983$$

(1) $$\arg(60) = \frac{\pi}{2} - \tan^{-1}(12) - \tan\left(\frac{0.2}{0}\right)$$
$$= \frac{\pi}{2} - 1.4877 - \frac{\pi}{2} = -1.488 \qquad (= -85.23^\circ)$$

Q3.

The measured force is therefore :

(1) $$F_M(t) = 25\left\{1.088 \sin\left(20t + \cancel{0.096}\ 0.170\right) + \frac{4.983}{3} \sin(60t - 1.488)\right\}$$

And the dynamic error :

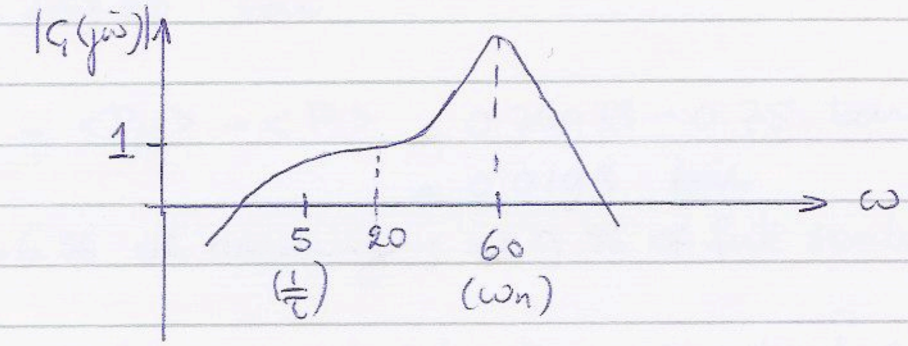
(1) $$E(t) = F_M(t) - F(t)$$

$$= 25\left\{1.088 \sin\left(20t + \cancel{0.096}\ 0.170\right) - \sin(20t)\right\}$$

(1) $$+ \frac{25}{3}\left\{4.983 \sin(60t - 1.488) - \sin(60t)\right\}.$$

(7 punten in totaal voor onderdeel a)

b)



Improving the system responce for the situation of a)

(1) most important : 1. increase natural frequency to well beyond 60 $\frac{rad}{sec}$; say 300 $\frac{rad}{sec}$

(1) 2. increase the damping ratio to close to the optimal value of $\zeta \sim 0.7$.

(1) less important : 3. increase the charge amplifier time constant ; say $\tau' = 1$ sec.

(3 pnt. totaal voor b)

Q4. $i = a + b \cdot P + c P^2 + d(T - T_0) P$

$P_M = \alpha + \beta (1 + \gamma \cdot i)^{1/2}$

a). for the accuracy, evaluate the mean value of the output P_M:

$$\langle i \rangle = (4 + 11 \cdot 0.75 + 5(0.75)^2 + 0.5(21-21) \cdot 0.75) \, mA$$
$$= 4 + 8.25 + 2.8125 + 0 \quad mA$$
$$= 15.0625 \ mA$$

2

$$\langle P_M \rangle = (-1.1 + 0.64(1 + 0.5 \cdot 15.0625)^{1/2}) \text{ bar}$$
$$= (-1.1 + 0.64 \cdot 2.92083) \quad \text{bar}$$
$$= 0.76933 \quad \text{bar}$$

$\rightarrow$ Error $= \langle P_M \rangle - \langle P \rangle = 0.76933 - 0.75$ bar
$= 0.0193$ bar

(= 2.6 % of reading ; = 1.9 % of full scale).

for the precision, evaluate the standard deviation of the output:

$$\sigma_i^2 = \left(\frac{\partial i}{\partial a}\right)^2 \sigma_a^2 + \left(\frac{\partial i}{\partial b}\right)^2 \sigma_b^2 + \left(\frac{\partial i}{\partial c}\right)^2 \sigma_c^2 + \left(\frac{\partial i}{\partial d}\right)^2 \sigma_d^2 +$$
$$\left(\frac{\partial i}{\partial T_0}\right)^2 \sigma_{T_0}^2 + \left(\frac{\partial i}{\partial T}\right)^2 \sigma_T^2 + \left(\frac{\partial i}{\partial p}\right)^2 \cdot \sigma_p^2$$
$$= 1^2 \cdot \sigma_a^2 + P^2 \cdot \sigma_b^2 + P^4 \sigma_c^2 + (T - T_0)^2 P^2 \cdot \sigma_d^2$$
$$+ (-d \cdot P)^2 \sigma_{T_0}^2 + (d \cdot P)^2 \sigma_T^2 + (b + 2cp + d(T - T_0))^2 \sigma_p^2$$

Q4 continued...

$$\sigma_i^2 = (3\,10^{-3})^2 + (0.75)^2 (5\,10^{-3})^2 + (0.75)^4 (10^{-4})^2 + 0.\sigma_d^2$$

$$+ (d \cdot P)^2 \cdot 0 + (0.5 \cdot 0.75)^2 \cdot (0.1)^2 + 0$$

$$= 9\,10^{-6} + 0.5625 \cdot 25 \cdot 10^{-6} + 0.3164\,10^{-8} + 0.1406\,10^{-2}$$

$$= 0.00143 \quad mA^2$$

$$\sigma_P^2 = \left(\frac{\partial P}{\partial \alpha}\right)^2 \sigma_\alpha^2 + \left(\frac{\partial P}{\partial \beta}\right)^2 \sigma_\beta^2 + \left(\frac{\partial P}{\partial \gamma}\right)^2 \sigma_\gamma^2 + \left(\frac{\partial P}{\partial i}\right)^2 \sigma_i^2$$

$$= 1 \cdot \sigma_\alpha^2 + (1+\gamma \cdot i)\,\sigma_\beta^2 + \frac{\beta^2}{4(1+\gamma i)}\sigma_\gamma^2 + \frac{(\beta\gamma)^2}{4(1+\gamma i)}\sigma_i^2$$

$$= 10^{-8} + (1 + 0.5 \cdot 15.0625) \cdot (5\,10^{-4})^2 + \frac{0.64^2}{4(1+0.5 \cdot 15.0625)} (10^{-3})^2$$

$$+ \frac{0.64^2 \cdot 0.25}{4(1+0.5 \cdot 15.0625)} \cdot (1.43\,10^{-3})^2 \quad bar^2$$

$$= 10^{-8} + 8.3125 \cdot 25 \cdot 10^{-8} + 0.0120\,10^{-6} + 0.0030 \cdot 2.045\,10^{-6} \quad bar^2$$

$$= 10^{-8} + 207.8\,10^{-8} + 12 \cdot 10^{-9} + 6.13\,10^{-9} \quad bar^2$$

$$= 2.11\,10^{-6} \quad bar^2$$

2 $\rightarrow \sigma_P = 1.45\,10^{-3}$ bar (= 0.19 % of reading ;
= 0.15 % of full scale)

Q4 continued...

b) for $P = 0$ bar the output does not change with temperature, Temperature is therefore a modifying input, not an interfering input.
(2)

c). Short from implementing a completely different design, one could consider the following:

1. program the μP with the inverse equation of the Baration. The coeff. can be determined by a calibration procedure. This is the subject of question d).
(2)
2. Temperature stabilisation at 21°C. Passively, or better actively.
3. Input the measurement of the environmental temperature to the μP and use it to correct the measurement of the pressure.

d). Note that the equation $P_M = \alpha + \beta(1+\gamma \cdot i)^{1/2}$

is/should be the inverse of the equation for i:
$i = a + b p + c p^2 + d \cdot (T - T_0) \cdot p$. Since we may take $T = 21°C = T_0$, the last term reduces to zero (instead of modifying b).
Thus:

$$i = a + b.P + c P^2$$

$$\rightarrow \quad P = -\frac{b}{2c} \pm \frac{1}{2c}\left(b^2 - 4c(a-i)\right)^{1/2}$$

$$= -\frac{11}{10} \pm \frac{1}{10}\left(41 + 20 \cdot i\right)^{1/2}$$

Q4 continued...

d).
$$P = -1.1 \pm 0.1 \cdot \sqrt{41} \left(1 + \frac{20}{41} \cdot i\right)^{1/2}$$

2

$$\rightarrow \begin{cases} \alpha = -1.1 \\ \beta = \sqrt{41}/10 = 0.640312 \\ \gamma = 20/41 = 0.487805 \end{cases}$$

if these parameters are used, instead of those given in Table 4.1, the mean value of the output will exactly equal the input pressure, provided the temperature $T = T_0 = 21\,°C$.

Q5

a). $\Delta V_u = \Delta i \cdot (R_S \,//\, Z_{C_S} \,//\, Z_{C_c} \,//\, R_L) =: \Delta i \cdot Z_{//}$

$$\frac{1}{Z_{//}} = \frac{1}{R_S} + \frac{1}{Z_{C_S}} + \frac{1}{Z_{C_c}} + \frac{1}{R_L}$$

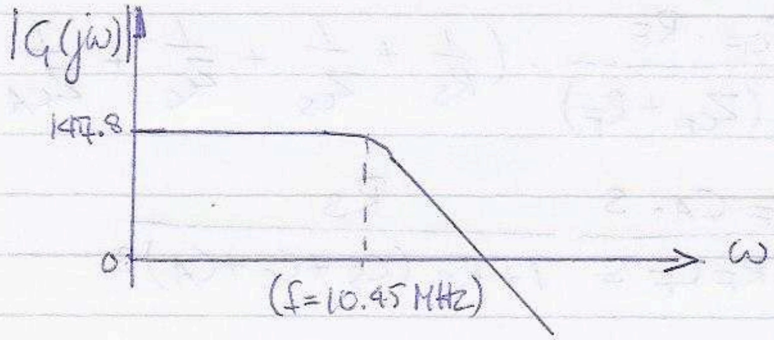
$$= \frac{1}{R_S} + \frac{1}{R_L} + (C_S + C_c)\cdot s$$

$$Z_{//} = \frac{R_S R_L}{(R_S + R_L) + R_S R_L (C_S + C_c) s}$$

1 $$G(s) = Z_{//} = \frac{R_S R_L / (R_S + R_L)}{1 + \frac{R_S R_L}{R_S + R_L} (C_S + C_c)\cdot s}$$

b)

1



a). $\tau \equiv \frac{R_S R_L}{R_S + R_L} (C_S + C_c) = \frac{150 \cdot 10^4}{150 + 10^4} \cdot (3 + 100) 10^{-12}$ sec.

$= 147.8 \cdot 103\, 10^{-12}$ sec $= 1.522\, 10^{-8}$ sec

$\omega_n = \frac{1}{\tau} = 6.569\, 10^7$ rad/sec $\rightarrow f = 1.045\, 10^7$ Hz

$= 10.45$ MHz

$\cancel{G(j\cdot \frac{63.7}{2\pi} 10^6) = G(j\, 1.013\, 10^6)} =$

a) $f = 63.7\ \text{MHz} \rightarrow \omega = 2\pi f = 400\ 10^6\ \text{rad/sec}$

$\rightarrow \omega\tau = 4 \cdot 10^8 \cdot 1.522\ 10^{-8} = 6.10$

3 $G(j\omega) = \dfrac{147.8}{1 + j \cdot 6.10}$

$|G(j\omega)| = \dfrac{147.8}{\sqrt{1+6.1^2}} = 23.9$

2 c). $20 \log \left(\dfrac{1}{\sqrt{1+6.1^2}} \right) = -15.8\ \text{dB}$

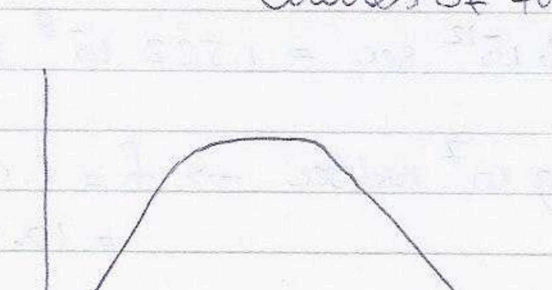
d). $\dfrac{\Delta V_u}{\Delta i} = -\dfrac{Z_F}{Z_{C_A}} \cdot Z'_{/\!/}$ with $Z_F = (Z_{C_F} /\!/ R_F)$

$Z'_{/\!/} = (R_S /\!/ Z_{C_S} /\!/ Z_{C_C} /\!/ Z_{C_A})$

$$G(s) = -\frac{Z_{CF} \cdot R_F}{Z_{CA}(Z_{CF} + R_F)} \cdot \left(\frac{1}{R_S} + \frac{1}{Z_{C_S}} + \frac{1}{Z_{C_C}} + \frac{1}{Z_{CA}} \right)^{-1}$$

3 $$= -\frac{R_F C_A \cdot s}{1 + R_F C_F s} \frac{R_S}{1 + R_S (C_S + C_C + C_A) s}$$

$G(j \cdot 4 \cdot 10^8) = - \ldots$ Oops! forgot to give the values of the op-amp components!



advantage may be a reduction of low-freq. noise contributions and the capability of the opamp to drive higher loads.